# HW4 Solutions

## § 3.1 – 8

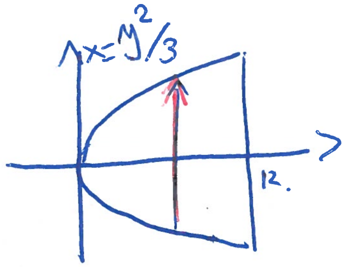
(a) $$\int_y^{y^2} (x-y)\,dx = \left(\frac{x^2}{2} - yx\right)\Big|_y^{y^2} = \frac{y^4}{2} - y^3 - \frac{y^2}{2} + y^2$$

$$= \frac{y^4}{2} - y^3 + \frac{y^2}{2}$$

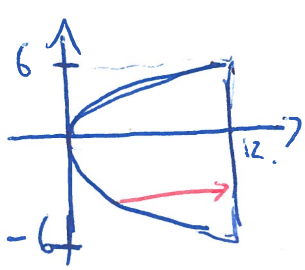
(b) $$\int_{-1}^{1}\int_y^{y^2} (x-y)\,dx\,dy = \frac{1}{2}\int_{-1}^{1} y^4 - \underbrace{\cancel{2y^3}}_{\text{odd}} + y^2\,dy = \frac{1}{2}\left(\frac{y^5}{5} - \cancel{\frac{2y^4}{2}} + \frac{y^3}{3}\right)\Big|_{-1}^{1}$$

$$= \frac{1}{2}\left(\frac{1}{5} + \frac{1}{3} + \frac{1}{5} + \frac{1}{3}\right) = \frac{1}{5} + \frac{1}{3} = \boxed{\frac{8}{15}}$$

## § 3.1 – 14




$$\boxed{A = \int_0^{12}\int_{-\sqrt{3x}}^{\sqrt{3x}} 1\,dy\,dx}$$

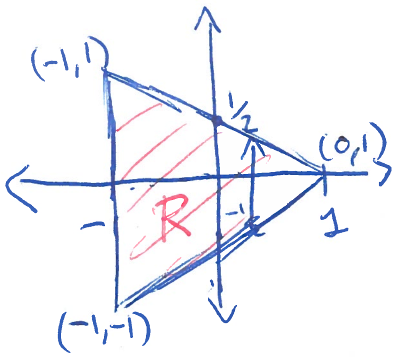



$$\boxed{A = \int_{-6}^{6}\int_{y^2/3}^{12} 1\,dx\,dy}$$

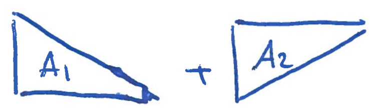
So $$A = \int_{-6}^{6} 12 - \frac{y^2}{3}\,dy = \left[12y - \frac{y^3}{9}\right]_{-6}^{6} = 144 - 48 = \boxed{96}$$

## § 3.1 − 22

$$A = \int_{-1}^{1} \int_{(x-1)/2}^{(1-x)/2} dy\,dx.$$




$$\boxed{A = \int_{-1}^{0} \int_{-1}^{2y+1} dx\,dy + \int_{0}^{1} \int_{-1}^{1-2y} dx\,dy}$$

(we split 

)

$\boxed{\text{note } A = 2}$

## § 13.2 − 6

(a) $$\int_{-\pi/2}^{\pi/2} \int_{0}^{\pi} \sin x \cos y \, dx\,dy = \int_{-\pi/2}^{\pi/2} \cos y \left(-\cos x\right)\Big|_0^{\pi} dy$$

$$= \int_{-\pi/2}^{\pi/2} 2\cos y\,dy = 2(\sin y)\Big|_{-\pi/2}^{\pi/2} = \boxed{4}$$

(b) $$\int_{0}^{\pi} \int_{-\pi/2}^{\pi/2} \sin x \cos y \, dy\,dx \leftarrow \text{Same thing!}$$

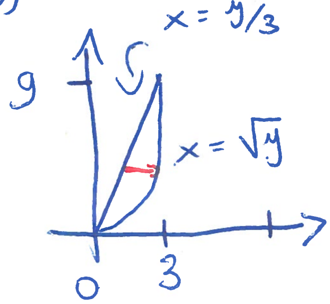
## § 13.2 − 10

(a) $$\int_0^9 \int_{y/3}^{\sqrt{y}} xy^2\,dx\,dy = \int_0^9 y^2 \left(\frac{x^2}{2}\right)\Big|_{y/3}^{\sqrt{y}} dy$$

$$= \int_0^9 \frac{y^3}{2} - \frac{y^4}{18}\,dy = \frac{y^4}{8} - \frac{y^5}{90}\Big|_0^9 = \frac{9^4}{8} - \frac{9^5}{9 \cdot 10} = 9^4\left(\frac{1}{8} - \frac{1}{10}\right)$$

$$= \frac{9^4}{40} = \boxed{\frac{6561}{40}}$$

(b)

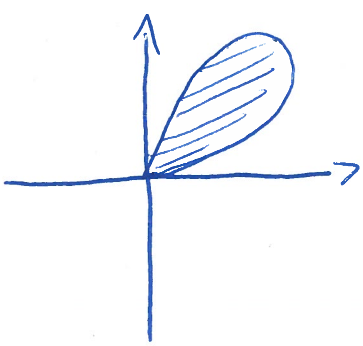



$$\boxed{I = \int_0^3 \int_{x^2}^{3x} xy\, dy\, dx}$$

## §13.2 −24

$$f_{Avg} = \frac{\int_{-\pi/2}^{\pi/2}\int_0^{\pi} \sin x \cos y\, dx\, dy}{\int_{-\pi/2}^{\pi/2}\int_0^{\pi} dx\, dy} = \boxed{\frac{4}{\pi^2}}$$

## §13.3 −6



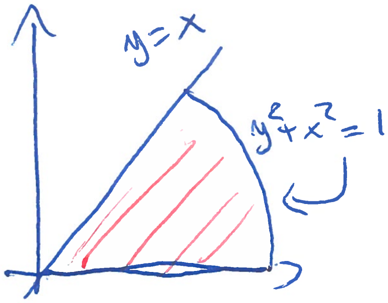
$$I = \int_0^{\pi/2}\int_0^{\sin 2\theta} 4\, R\, dR\, d\theta.$$

first quadrant

$$I = \int_0^{\pi/2} 2\, R^2 \Big|_0^{\sin 2\theta} d\theta = \int_0^{\pi/2} 2 \sin^2 2\theta\, d\theta$$

$$= \int_0^{\pi/2} 1 - \cos 4\theta\, d\theta = \left(\theta - \frac{\sin 4\theta}{4}\right)\Big|_0^{\pi/2} = \boxed{\pi/2}$$

# § 13.3 – 101




$$I = \int_0^{\pi/4}\int_0^1 \frac{x-y}{x+y}\, R \, dR\, d\theta.$$

$x = x(R,\theta)$
$y = y(R,\theta)$

so $I = \displaystyle\int_0^{\pi/4}\int_0^1 \frac{R^2(\cos\theta - \sin\theta)}{R\cos\theta + \sin\theta}\, dR\, d\theta = \boxed{\int_0^{\pi/4}\int_0^1 \frac{R(\cos\theta - \sin\theta)}{\cos\theta + \sin\theta}\, dR\, d\theta}$

Let's compute $I = \displaystyle\int_0^{\pi/4} \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\left(\frac{R^2}{2}\right)\Bigg|_0^1 d\theta = \frac{1}{2}\int_0^{\pi/4} \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\, d\theta.$

let $u = \cos\theta + \sin\theta \qquad du = (\cos\theta - \sin\theta)\, d\theta.$

So $I = \displaystyle\frac{1}{2}\int_1^{\sqrt{2}} \frac{1}{u}\, du = \frac{1}{2}\ln(u)\Big|_1^{\sqrt{2}} = \frac{1}{2}(\ln\sqrt{2} - \ln(1))$

$$\boxed{I = \frac{\ln(2)}{4}}$$