HW 1 Sols

Math 253:928

2008 S1

§10.1 – 10

$x^2+y^2+z^2+4x-2y-4z+4=0$ complete squares.

$x^2+4x+4+y^2-2y+1+z^2-4z+4+4=0+9$

$(x+2)^2+(y-1)^2+(z-2)^2=5=(\sqrt{5})^2$

Center is $(-2,1,2)$ radius is $\sqrt{5}$ //

§10.2 – 20

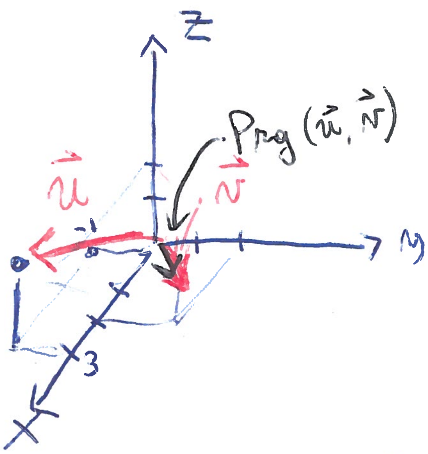
Vectors must be parallel.

ie $\vec{v}=c\cdot\vec{u}$ for some $c\in\mathbb{R}$, $c\geq 0$

§10.3 – 26

$\operatorname{Proj}(\vec{u},\vec{v})=\frac{\langle\vec{u},\vec{v}\rangle\cdot\vec{v}}{\|\vec{v}\|^2}=\frac{6-2+2}{3\cdot 3}\cdot(2,2,1)$

$=\left(\frac{4}{3},\frac{4}{3},\frac{2}{3}\right)$



§10.3 – 32

$\vec{u}=\operatorname{Proj}(\vec{u},\vec{v})+\left(\vec{u}-\operatorname{Proj}(\vec{u},\vec{v})\right)$

$\parallel$ to $\vec{v}$ $=\left(\frac{4}{3},\frac{4}{3},\frac{2}{3}\right)+\left(\frac{5}{3},\frac{-7}{3},\frac{4}{3}\right)$ $\perp$ to $\vec{v}$.

§ 10.4 − 30

let $a = (0,0,0)$

$b = (-1,2,-8)$

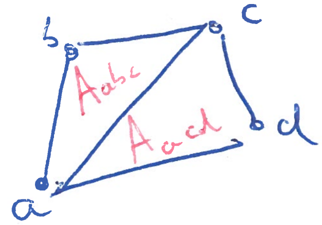
$c = (2,1,1)$

$d = (1,-1,5)$

SANITY CHECK

a, b, c, d lie on a plane

$\frac{1}{3} c + \left(\frac{-5}{3}\right) d = b$



$$A = A_{abc} + A_{acd}$$

$$A_{abc} = \frac{1}{2} \|\vec{ab} \times \vec{ac}\|$$

$$= \frac{1}{2} \|(-1,2,-8) \times (2,1,1)\|$$

$$= \frac{1}{2} \|(10,-15,-5)\| = \frac{5}{2}\sqrt{14}$$

$$A_{acd} = \frac{1}{2} \|\vec{ad} \times \vec{ac}\|$$

$$= \frac{1}{2} \|(1,-1,5) \times (2,1,1)\|$$

$$= \frac{1}{2} \|(-6,9,3)\| = \frac{3}{2}\sqrt{14}$$

$$A = 4\sqrt{14} \approx 14{,}96$$

§ 10.4 − 31

$$A = \vec{u} \cdot (\vec{v} \times \vec{w})$$

$$\vec{v} \times \vec{w} = (1,2,3) \times (-1,0,1) = (2,2,-2)$$

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = (1,1,1) \cdot (2,2,-2) = 2$$

$$A = 2$$

§10.5 - 12)

$l_1(t) = (0,-2,1) + t(1,2,1) \qquad l_2(s) = (2,2,3) + s(1,-1,2)$

solve $l_1(t) = l_2(s)$.

$\iff t(1,2,1) = (2,4,2) + s(1,-1,2)$

i.e.: $t = 2+s$
$2t = 4-s$
$t = 2+2s$

sol $\boxed{\begin{array}{l} s=0 \\ t=2 \end{array}}$

so intersection is $(2,2,3)$.

direction is $(1,2,1) \times (1,-1,2) = (5,-1,-3)$

so $\boxed{\vec{r}(t) = (2,2,3) + t(5,-1,-3)}$ vector

$\cancel{\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}}$

$\boxed{\begin{array}{l} x(t) = 2+5t \\ y(t) = 2-t \\ z(t) = 3-3t. \end{array}}$ Parametric

$\boxed{\dfrac{x-2}{5} = \dfrac{y-2}{-1} = \dfrac{z-3}{-3}}$ SYMMETRIC.

note: don't worry about the names of the forms. Give full marks if they at least got one right.

§10.6 - 10

find two non-parallel vectors and compute a normal using cross product.

$\vec{u} = (5,3,8) - (3,3,3) = (2,0,5)$

$\vec{v} = (6,4,9) - (3,3,3) = (3,1,6)$

$\vec{n} = \vec{u} \times \vec{v} = (-5,3,2)$

the standard form yields:

$$\boxed{-5(x-3) + 3(y-3) + 2(z-3) = 0.}$$

the general form gives:

$$\boxed{-5x + 3y + 2z = 0}$$

Note: Give full marks for getting any of the previous two forms.