1. Consider the surface described by the equation $e^{xyz-1} + 3 = x^2 + y^2 + z^2$.

4 marks

(a) Find a function $F(x, y, z)$ such that (a, b, c) is on the surface if and only if we have $F(a, b, c) = 0$. Compute the gradient of F.

4 marks

(b) Give the equation of the tangent plane to the surface at $(x_0, y_0, z_0) = (1, 1, 1)$ in the form $x + by + cz + d = 0$. Note that we require the coefficient next to x to be 1.

2. The sugar concentration in an infinite 3D compost bin is given by the equation $S(x, y, z) = (x + y + 2z)^2$. A fruit fly is at position $(1, 1, 1)$.

2 marks

(a) Compute the directional derivative of S at $(1, 1, 1)$ in the direction of $\vec{u} = (-1, 0, 1)$.

2 marks

(b) The fruit fly is feeling a little hungry. In what (unit) direction should the fly move if it wishes to increase the concentration of sugar in the fastest possible way?

2 marks

(c) The fly is now happy with the amount of sugar in its position. Give a (unit) direction in which the fly could move if it wishes to keep the concentration of sugar constant.

6 marks

3. (a) A differentiable function $z = f(x, y)$ is unknown, but an alien supercomputer gave us precise values of $f(x, y)$ and its derivatives on points A, B, C and D.

point	f	f_x	f_y	f_{xx}	f_{yy}	f_{xy}
A	1	0	0	1	0	-5
B	1	0	-2	3	8	4
C	2	0	0	3	3	-2
D	2	0	0	3	3	6

For points A, B, C and D determine whether they are a local minimum, local maximum, a saddle point, or none of the above.

- A is:
- B is:
- C is:
- D is:

2 marks

(b) **(Bonus marks)** Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function such that $f(1, 0) = f(0, 0) = 0$. Show that there exists $\langle a, b \rangle$ such that $\nabla f(a, b)$ is orthogonal to $\langle 1, 0 \rangle$.
Hint: Define $g(t) = f(t, 0)$. Combine the 1D mean value theorem and the chain rule to conclude.

This page has been left blank for your rough work and calculations.